

LESSON SUMMARY

CXC CSEC MATHEMATICS

UNIT TWO:
COMPUTATION

Lesson

2

Manipulation of Real Numbers

Textbook: Mathematics, A Complete Course by Raymond Toolsie, Volume 1'.

(Some helpful exercises and page numbers are given throughout the lesson e.g. Ex 3a page 44)

INTRODUCTION

This lesson aims to develop some computational skills that involve the manipulation of real numbers. These involve computations with fractions and percentages. These numbers and the sets they belong to were introduced in lesson 1. The skills presented here are needed to solve real life problems which will be encountered throughout the course.

OBJECTIVES

At the end of this lesson you will be able to:

- a) Perform basic operations with real numbers;
- b) Solve problems involving real numbers;
- c) Convert from percentages to decimals and vice versa;
- d) Calculate fractions or percentages of quantities;
- e) Express one quantity as a fraction or percentage of another;



2.1 Use of Calculators

The use of calculators will be developed throughout the course. The following are some uses that will be helpful in this lesson. Remember that different calculators may have different procedures in evaluating computational task. In this case the manual will prove to be very helpful.

Basic operations: These are done with the $+$ $-$ \times \div keys.

Powers of numbers: These can be evaluated using the x^2 x^3 and y^x keys. Some calculators have $^{\wedge}$ instead of y^x .

Example: To evaluate $3^4 + 2$ on your calculator simply enter 3 y^x 4 $+$ 2 $=$.

Operation with fractions: To enter fractions or mixed numbers on your calculator use the a^b/c or fraction key.

Example: To enter $3\frac{1}{2}$, press 3 a^b/c 1 a^b/c 2 . To convert it to decimal simply press the fraction key again.

Operations with decimals: Decimals are entered using the \cdot or decimal key.

Working out percentages: To enter percentages you may have to press the 2^{nd} function followed by the $\%$ key.



ACTIVITY 1

Evaluate the following with the aid of a calculator. Remember to press equal or enter to get the answer.

1. 2^5

2. $2\frac{2}{3} + \frac{5}{6}$

3. $8.09 - 3.004$

4. $25\% \times 607$

5. $\frac{3}{10} \div \frac{2}{5}$

2.2 Conversion of units (This will be done in lesson six)

2.3 Order of operations

Use BODMAS to carry out arithmetic operations. That is, work out brackets first, then division or multiplication, followed by addition or subtraction. (Ex 3a page 44)



Find the value of $11 - 12 \div 4 + 3(6 - 2)$

$$11 - 12 \div 4 + 3 \times 4$$

$$11 - 3 + 12$$

$$8 + 12 = 20$$

2.4 Fractions, Decimals, Percentages

Operation with fractions

Adding or subtracting fractions

If the denominators are different find the L.C.M of the denominators.

Example: Simplify $\frac{3}{5} + \frac{4}{7}$.

Solution: $\frac{3}{5} + \frac{4}{7}$

$\frac{3}{5} = \frac{3 \times 7}{5 \times 7} = \frac{21}{35}$ This means equivalent to.

$\frac{4}{7} = \frac{4 \times 5}{7 \times 5} = \frac{20}{35}$

$\frac{21}{35} + \frac{20}{35} = \frac{41}{35} = 1\frac{6}{35}$

35
L.C.M of 5 and 7.

Multiplying fractions

Simply multiply the numerators and then multiply the denominators. First remember to divide the numerators and denominators by any common factors so that the answer is in its simplest form.

Example: Determine the value of each of the following $\frac{3}{5} \times \frac{8}{9}$

Solution: $\frac{3}{5} \times \frac{8}{9} = \frac{1 \times 8}{5 \times 3} = \frac{8}{15}$

Dividing by fractions

To divide by a fraction invert the fraction and multiply.

Example: Calculate $\frac{9}{28} \div \frac{3}{14}$

Solution: $\frac{9}{28} \div \frac{3}{14} = \frac{9}{28} \times \frac{14}{3}$

$$\frac{9}{28} \times \frac{14}{3} = \frac{3}{2} = 1\frac{1}{2}$$

Operations with mixed numbers

You may convert to improper fractions and then perform any operation that is required.

Example: simplify $8\frac{3}{4} - 5\frac{1}{3} = \frac{35}{4} - \frac{16}{3}$.

Operations with decimals

Addition or subtraction of decimals

Put the point under point when adding or subtracting decimals.

Example: Write down the value of $4.317 - 0.015$

Solution:
$$\begin{array}{r} 4.317 \\ - 0.015 \\ \hline 4.302 \end{array}$$

Multiplying decimals

Multiplying or dividing decimals by powers of 10 e.g. 10, 100, 1000 etc.

To multiply simply move the point the same number of places to the right as the number of zeros in the power of 10. To divide move the point the required number of places to the left.

Example: Determine the value of 37.58×1000

Solution: move the point 3 places to the right. Put zeros as required.

$37\overset{\curvearrowright}{5}\overset{\curvearrowright}{8}\overset{\curvearrowright}{0} = 37580$
zero inserted to jump.

Multiply by a number that is not a power of 10

Multiply as normal but remember the answer must have the same number of decimal places as in the original item.

Example: Multiply 3.5 by 1.5

Solution: 3.5

$$\begin{array}{r}
 \times 1.5 \\
 \hline
 350 \\
 +175 \\
 \hline
 525
 \end{array}$$

The item has in total 2 decimal places therefore the answer must have 2 decimal places also.
 Answer = 5.25

Division by a decimal

Convert the decimal to a whole number by shifting the point the required number of places to the right.

Example: Evaluate $56.86 \div 0.4$

Solution:

Shifting the point one place in the divisor will turn it into a whole number therefore you have to shift the point one place in the number being divided also.

$$\begin{array}{r}
 568.6 \div 0.4 \rightarrow 5686 \div 4
 \end{array}$$

▲

$$\begin{array}{r}
 1421.5 \\
 4 \overline{) 5686} \\
 \underline{- 4} \\
 16 \\
 \underline{- 16} \\
 08 \\
 \underline{- 08} \\
 06 \\
 \underline{- 04} \\
 20 \leftarrow \text{add zeros when you have no more digits to bring down} \\
 \underline{- 20} \\
 0
 \end{array}$$

put a point where you meet it in the calculation

Mixed operations with fractions or decimals

Follow BODMAS,

Example: Calculate the exact value of $\frac{45.37 - 24.16}{13.74 + 7.26}$

Solution: $\frac{21.21}{21} = 1.01$

Converting fractions and percentages to decimals

Recall a percentage is a fraction with a denominator of 100, e.g. $25\% \equiv \frac{25}{100}$. To convert any fraction to a decimal, divide the numerator by the denominator.

Example: convert $\frac{4}{5}$ to a decimal.

Solution:
$$\begin{array}{r} 0.8 \\ 5 \overline{) 40} \\ \underline{- 40} \\ 0 \end{array}$$
 5 cannot divide 4 therefore put a zero to make it 40, and put a point in the answer.

Therefore $\frac{4}{5} \equiv 0.8$ (you may use a calculator)

When converting percentages to decimals just remove the percent sign and move the point two places to the left.

Example: $25\% \equiv 0.25$

Changing a decimal to fraction

Put one for the point and zero for every digit after the decimal point. Therefore $\frac{0.8}{10} = \frac{8}{10} = \frac{4}{5}$

A fraction or percentage of a quantity.

To determine a fraction or percentage of any quantity simply multiply by the fraction or percentage.

Example: find $\frac{3}{5}$ of 500 = $\frac{3}{5} \times 500 = 300$

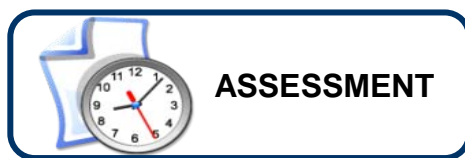
The solution is 300. (you may use a calculator.)

Expressing one quantity as a fraction or percentage of another.

To express one quantity as a fraction of another, simply put the quantity as the numerator and the other as the denominator, then reduce this fraction to its lowest term. If a percentage is required just multiply the fraction by 100.

Example: John owed Ian \$500 he paid him \$65. What percentage of the debt did John pay?

Solution: $\frac{65}{500} = \frac{13}{100}$, for percentage multiply by 100, we get 13%.



(Past CXC items)

1. **Using a calculator**, or otherwise, determine the EXACT value of:

(i) $(1.7)^2 + (1.3)^2$

(ii) $\frac{4.8 + 6.9}{1.3 \times 0.2}$

2. Calculate the exact value of

$$\frac{3\frac{1}{4} - 2\frac{1}{3}}{1\frac{5}{6}}$$

CONCLUSION

We have looked at a number of computational tasks involving real numbers. The key to develop in this area is to practice. Another important skill is the ability to approximate the results of computations. In lesson three we will be looking at approximation and expressing quantities in standard form.